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Application of the weak-coupling model for calculating the binding energies of neutron-rich A $\sim$ 32 nuclei

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Abstract. The presence of an anomaly in binding energies for the "island of inversion" centered at $Z = 11$, $N = 21$ is obtained by comparison of macroscopic binding energies and experiment. The macroscopic calculations were done with a mass formula deduced from the reformulation of the liquid-drop model of Myers *et al.*; this formula is described in detail such as its predictions for binding energies of neutron-rich $A = 29$–44 nuclei. These calculations have indicated the presence of anomalies in the "island of inversion". A weak-coupling approximation is applied to study this deformed region. The binding-energy values obtained using this model show the absence of these anomalies.

PACS. 21.10.Dr Binding energies and masses – 27.30.+t $20 \leq A \leq 38$

1 Introduction

The measurements made by Thibault *et al.* [1] on the $^{27-32}$Na have indicated that ^{31}Na and ^{32}Na were more bound than what was given by the theoretical predictions. These results have indicated the first signs of the inversion of shell ordering around $N = 20$. This phenomenon was observed as irregularities in the binding energies of neutron-rich $A \sim 32$ nuclei. Note that some of these nuclei are deformed in their ground state, and this deformation is due to the coexistence of the normal and intruder configuration.

Detraz *et al.* [2] have included in their study the Mg isotopes and found that the first excited state of ^{32}Mg lies at a low excitation energy of 885 keV which indicates a nuclear deformation.

Basing on shell model calculations, Chung *et al.* [3] showed that the binding energy of $N = 20$ Na and Mg isotones could not be understood by using shell model interactions with only the orbits of *sd* space.

Because of this, Poves *et al.* [4] carried out shell model calculations in the $\pi d_{5/2}^{Z-8} d_{3/2}^{N-18} (f_{7/2}, p_{3/2})^2$ space and examined the properties of E_2 for the considered nuclei.

Brown *et al.* [5] developed an interaction in the $\pi d_{5/2}^{Z-8} d_{3/2}^{N-18} (f_{7/2}, p_{3/2})^2$ space designated WBMB. They have employed it for the calculations of neutron-rich $A = 29$–44 nuclei.

For exploring the $2\hbar\omega$ binding energy systematic for a large range of Z and N, they carried out calculations using the weak-coupling model [6] based on $n\hbar\omega$ excitations. They found that it is a good approximation for the full calculations.

Further results in this region have been obtained by Nummela *et al.* [7,8] who have performed beta decay studies for determining the level structure of ^{34}Si ($N = 20$), the first information on level structure of the $N = 21$ nuclei, ^{35}Si and ^{33}Mg, and have corrected the position of the level $(3/2)^-$ in ^{35}Si.

2 Macroscopic calculations

2.1 The mass formula

The aim is to develop a mass formula for calculating the binding energies of neutron-rich $A = 29$–44 nuclei. To do so we have reformulated the liquid-drop model of Myers *et al.* [9] taking the Weizsäcker mass formula as a starting point and basing on the relative neutron excess I.

After introducing the relative neutron excess in the Weizsäcker formula, we obtain

$$\begin{aligned} BE &= a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{\frac{1}{3}}} - a_a \frac{(N-Z)^2}{A} \pm \delta \\ &= a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{\frac{1}{3}}} - a_a I^2 A \pm \delta \\ &= a_v (1 - k_v I^2) A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{\frac{1}{3}}} \pm \delta \,. \end{aligned} \tag{1}$$

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As we see, the form of the volume term is not like that of the surface term although these two terms depend on the geometry of the nucleus (volume and surface). So because these two terms play the same role, they must have the same form, so we write the surface term as the volume term and the equation becomes

$$BE = a_v(1 - k_v I^2)A - a_s(1 - k_s I^2)A^{\frac{2}{3}} - E_c \pm \delta\,. \quad (2)$$

The electrostatic energy is [10]

$$E_c = \left[\frac{3}{5}\frac{e^2}{r_0}\frac{Z^2}{A^{1/3}} - \frac{\pi^2}{2}\frac{e^2}{r_0}\left(\frac{d}{r_0}\right)^2\frac{Z^2}{A}\right]. \quad (3)$$

We take for δ the corrected form of Myers *et al.* [11]:

$$\delta = \begin{cases} \left(\frac{12}{\sqrt{A}} - \frac{10}{A}\right), & Z \text{ and } N \text{ even}, \\ -\frac{10}{A}, & Z \text{ or } N \text{ odd}, \\ -\left(\frac{12}{\sqrt{A}} - \frac{10}{A}\right), & Z \text{ and } N \text{ odd}. \end{cases} \quad (4)$$

For $Z = N$ we see that I vanishes so we introduce the first correction on this mass formula, which is the Wigner term [11] given by

$$\Delta E_{\text{wig}} = 30(|I| + d), \quad (5)$$

where

$$d = \begin{cases} \frac{1}{A}, & Z \text{ and } N \text{ odd}, \quad Z = N, \\ 0, & \text{otherwise}. \end{cases} \quad (6)$$

Taking into account the movement of nucleons in the nucleus, we introduce the second correction which is the shell correction given by [10]:

$$E_{\text{shell}} = \alpha\left(\frac{F(N) + F(Z)}{\left(\frac{A}{2}\right)^{\frac{2}{3}}} - \beta A^{\frac{1}{3}}\right), \quad (7)$$

$$F(N) = \frac{3}{5}\frac{\left(M_i^{\frac{5}{3}} - M_{i-1}^{\frac{5}{3}}\right)}{(M_i - M_{i-1})}(N - M_{i-1}) - \frac{3}{5}\left(N^{\frac{5}{3}} - M_{i-1}^{\frac{5}{3}}\right), \quad \text{for } M_{i-1} < N < M_i, \quad (8)$$

$$F(Z) = \frac{3}{5}\frac{\left(M_i^{\frac{5}{3}} - M_{i-1}^{\frac{5}{3}}\right)}{(M_i - M_{i-1})}(Z - M_{i-1}) - \frac{3}{5}\left(Z^{\frac{5}{3}} - M_{i-1}^{\frac{5}{3}}\right), \quad \text{for } M_{i-1} < Z < M_i. \quad (9)$$

Here M_{i-1}, M_i are the magic numbers.

The mass formula becomes

$$BE = a_v(1 - k_v I^2)A - a_s(1 - k_s I^2)A^{\frac{2}{3}} - a_{c1}\frac{Z^2}{A^{\frac{1}{3}}} + a_{c2}\frac{Z^2}{A} + E_{\text{shell}} \pm \delta'\,, \quad (10)$$

where $a_v, k_v, a_s, k_s, a_{c1}, a_{c2}$ are parameters to be determined by a fit to the experimental binding energies of the nuclei:

$$k_v = a_a/a_v, \quad k_s = a_a/a_s, \quad I = (N - Z)/A.$$

The term δ' can be written as follows:

$$\delta' = \begin{cases} +\left(\frac{12}{\sqrt{A}} - \frac{10}{A}\right) - 30\left|\frac{N-Z}{A}\right|, & Z \text{ and } N \text{ even}, \\ -\frac{10}{A} - 30\left|\frac{N-Z}{A}\right|, & Z \text{ or } N \text{ odd}, \\ -\left(\frac{12}{\sqrt{A}} - \frac{10}{A}\right) - 30\left|\frac{N-Z}{A}\right|, & Z \text{ and } N \text{ odd}, \\ -\left(\frac{12}{\sqrt{A}} + \frac{20}{A}\right) - 30\left|\frac{N-Z}{A}\right|, & Z \text{ and } N \text{ odd}, \ Z = N. \end{cases} \quad (11)$$

2.2 Determination of the parameters

For deducing the parameters included in the formula, we have fitted the experimental masses (binding energies) of the nuclei [12]. We have excluded the masses without indication of precision errors because we need their errors in the fit, and supposed that $k_v = k_s = k$ in the formula. The procedure of the fit is as in ref. [10].

2.3 Results

The fit gives this series of parameters:

$a_v = 13.83\,\text{MeV}$, $k_v = 1.99$,
$a_s = 12.43\,\text{MeV}$, $k_s = 2.52$,
$a_{c1} = 0.72\,\text{MeV}$, $a_{c2} = 1.18\,\text{MeV}$,
$\alpha = 8.47\,\text{MeV}$, $\beta = 1.10$.

The results obtained for the binding energies of neutron-rich $A = 29$–44 nuclei are not satisfying compared to experiment, and have indicated anomalies when they were distributed on a (N, Z) chart. These anomalies are around the region $A \sim 32$ (fig. 1).

To solve this problem, we have performed weak-coupling calculations applied to particle-hole states in the mass region $A \sim 32$.

3 Weak-coupling calculations

The weak coupling is an interaction between the collective motion of nucleons in the same shell and the motion of the single particle. This model is applied for nuclei near sphericity.

The Hamiltonian is given by

$$H = H_1 + H_2 + H_{\text{int}}\,, \quad (12)$$

where H_1 is the Hamiltonian of the first group of particles, H_2 is the Hamiltonian of the second group of particles,

18	ldm	305.24	314.16	327.17	332.72	342.39	347.62
	exp	306.70	315.49	327.33	333.93	343.79	349.89
17	ldm	296.28	304.72	316.60	321.30	329.52	334.30
	exp	298.19	306.78	317.09	323.19	331.27	337.10
16	ldm	290.04	296.82	308.54	311.33	319.09	322.36
	exp	291.83	298.81	308.70	313.01	321.04	325.41
15	ldm	278.28	284.57	294.93	297.03	303.28	306.10
	exp	280.95	287.24	295.61	299.07	305.88	309.44
14	ldm	269.19	273.74	283.76	284.06	289.84	291.10
	exp	271.40	275.28	283.42	285.89	292.01	294.19
13	ldm	254.27	258.33	266.73	266.53	270.75	271.56
	exp	254.99	259.16	264.68	267.11	272.32	274.49
12	ldm	241.95	244.19	252.02	250.19	253.95	253.14
	exp	241.63	244.04	249.69	251.76	256.53	(256.80)
11	ldm	223.44	225.19	231.09	229.02	231.16	229.39
	exp	222.84	224.34	228.94	231.37	232.24	(233.31)
10	ldm	207.43	207.28	212.35	208.85	210.56	
	exp	206.89	208.24	212.11	(211.85)	(213.26)	
Z / N		18	19	20	21	22	23

Fig. 1. Distribution of experimental and calculated binding energies on a (N, Z) chart. ldm: liquid-drop model calculations, exp: experiment. The dashed lines surround the island of inversion region, the magic number 20 is selected by thick lines.

H_{int} is the Hamiltonian of the interaction between the two groups of particles.

In the B-F-Z model [13–16], the interaction is characterized by the following Hamiltonian:

$$H_{\text{int}} = \gamma N_1 \cdot N_2 + \beta T_1 \cdot T_2 \,, \tag{13}$$

where N_1 and N_2 are the numbers of particles in the two interacting levels; T_1 and T_2 are the isospins of the two interacting levels.

The diagonalisation of H is done by the diagonalisation of each term separately in the basis

$$\psi_{IM,TT_z} = \left[(l)^{n_1}_{j_1,T_1} \times (l)^{n_2}_{j_2,T_2} \right]_{IM,TT_z} . \tag{14}$$

In this case, the excitation energies are given by

$$E_x = \varepsilon_1 + \varepsilon_2 + \gamma n_1 n_2 + n_{1\pi} n_{2\pi} \varepsilon_{\text{C}} + \frac{\beta}{2} \Big[T\,(T+1) - T_1\,(T_1+1) - T_2\,(T_2+1) \Big] , \tag{15}$$

where ε_1 is the energy of the holes in the first level, ε_2 is the energy of the particles (neutrons) in the second level. (We deal with the neutron excitations, so Z is constant and these two energies are calculated from masses of the considered isotopes.) $n_{1\pi}$ and $n_{2\pi}$ are the number of protons in the two interacting levels. ε_{C} is the Coulomb interaction between the protons.

Note that this formula is applied when j_1 and/or j_2 is equal to zero.

In order to calculate the masses using this model, we have applied this formula on particle-hole configurations for the neutron-rich $A \sim 32$ nuclei.

We have studied nuclei having $Z = 10$–15, $N = 18$–23.

We have taken only the neutron excitations, so we neglect the term of the Coulomb energy and that of the isospin. We obtain the following formula:

$$E_x = \varepsilon_1 + \varepsilon_2 + n_1 n_2 \gamma \,. \tag{16}$$

Because the anomalies are following the variation of N for each Z constant (fig. 1), we have replaced the constant γ by a linear function in Z, following an idea of Warburton *et al.* [6].

In this case, the excitation energies are given by

$$E_x = \varepsilon_1 + \varepsilon_2 + n_1 n_2 \,(a_1 Z + a_2) \,. \tag{17}$$

And the masses are given by

$$M = M_c + \varepsilon_1 + \varepsilon_2 + n_1 n_2 \,(a_1 Z + a_2) \,, \tag{18}$$

where M_c is the mass of the core ($N = 20$) of the considered nucleus.

The binding energy is given by

$$BE = Z m_p + N m_n - M \,. \tag{19}$$

3.1 Determination of the parameters

We have determined the constants of this function by fitting the experimental excitation energies of neutron-rich $A \sim 32$ nuclei [8,12].

3.2 Results

The parameters of this function obtained after the fit are

$$a_1 = -33\,\text{keV}, \qquad a_2 = -8\,\text{keV}.$$

The results obtained for the binding energies of neutron-rich $A \sim 32$ nuclei are found in good agreement with experiment.

In fig. 2, we represent the experimental and calculated binding energies of the neutron-rich $A \sim 32$ nuclei. We observe for $Z = 12$ that when N increases, the binding energies calculated by the weak-coupling model (wcm) converge to the experimental values more than those calculated with the mass formula.

In the case of $Z = 11$ the wcm is better than the ldm but is still not very accurate especially for $N > 20$. In the case of $Z = 10$ the wcm is not very good for smaller values of N where the ldm is better.

The reason is the strong deformation in the island of inversion centered at $Z = 11$, $N = 21$ and the resistance of its nuclei to this deformation. Another possible reason is that for $Z = 10$ and $Z = 11$ (far from the shell closure $Z = 14$) there is a possible interaction between the holes in the proton shell and the excited neutrons.

We see that this model is a good approximation to study the binding energies for nuclei having great values of Z, and it is not better for small values of Z.

Figure 3 represents a distribution of experimental and calculated binding energies, where we can clearly see the absence of anomalies.

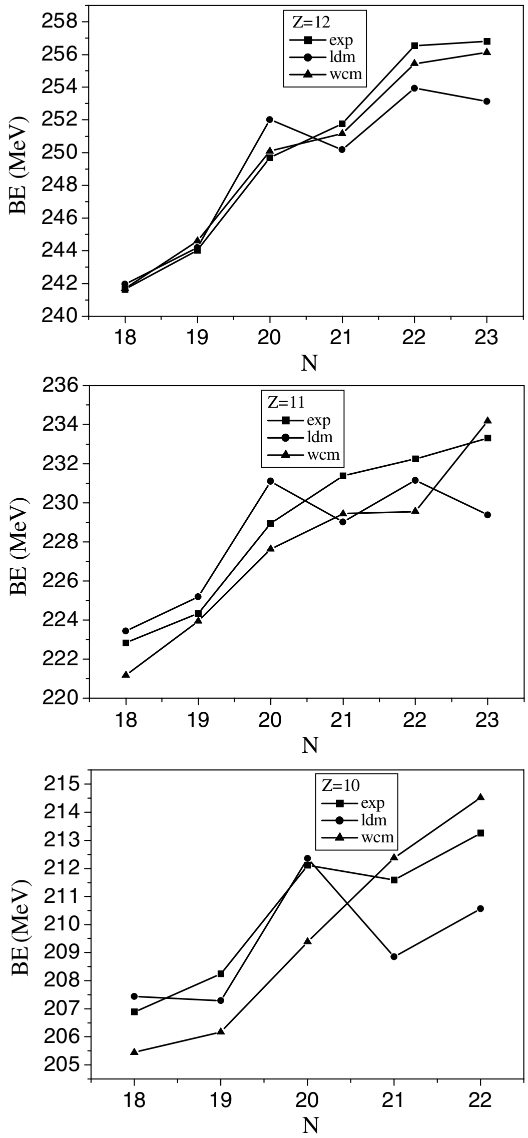

Fig. 2. Representation of experimental and calculated binding energies using the liquid-drop model (ldm) and the weak-coupling model (wcm) *versus* N.

4 Conclusion

In this work, we have calculated binding energies of the neutron-rich $A = 29$–44 nuclei using a mass formula deduced from the reformulation of the model of Myers *et al.* [9]. This reformulation was done by taking the Weizsäcker mass formula as a starting point, and basing on the relative neutron excess I. The values obtained using this formula are not in agreement with experiment, and when they were distributed on a (N, Z) chart they have indicated bad distribution in the $A \sim 32$ region.

For solving this problem, we have carried out weak-coupling calculations of binding energies of neutron-rich $A \sim 32$ nuclei. The values obtained are in agreement with experiment. Another distribution of these values on a (N, Z) chart shows the absence of anomalies. These results indicate that the weak-coupling model can be a good approximation for studying this deformed region.

Z		18	19	20	21	22	23
15	wcm	277.21	284.14	293.23	297.79	305.05	309.81
	ldm	278.28	284.57	294.93	297.03	303.27	306.10
	exp	280.95	287.24	295.61	299.07	305.88	309.43
14	wcm	267.96	273.60	281.87	284.99	291.19	294.40
	ldm	269.19	273.74	283.76	284.06	289.84	291.10
	exp	271.40	275.28	283.42	285.89	292.01	294.19
13	wcm	253.31	258.19	264.44	267.42	272.67	275.51
	ldm	254.27	258.33	266.73	266.53	270.75	271.56
	exp	254.99	259.16	264.68	267.11	272.32	274.49
12	wcm	241.69	244.60	250.08	251.16	255.44	256.12
	ldm	241.96	244.19	252.02	250.19	253.95	253.14
	exp	241.63	244.04	249.69	251.76	256.53	(256.80)
11	wcm	221.69	223.95	227.62	229.44	229.55	234.18
	ldm	223.44	225.19	231.09	229.02	231.16	229.39
	exp	222.84	224.34	228.94	231.37	232.24	(233.31)
10	wcm	205.44	206.18	209.38	212.37	214.51	
	ldm	207.43	207.28	212.35	208.85	210.56	
	exp	206.89	208.24	212.11	(211.58)	(213.26)	
Z / N		18	19	20	21	22	23

Fig. 3. Distribution of experimental and calculated binding energies. exp: experimental values, ldm: calculated values using the liquid-drop model, wcm: calculated values using the weak-coupling model. The dashed lines surround the island of inversion region, the magic number 20 is selected by thick lines.

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